NASA TN D-78

NASA TN D-78

7N-28
195521
29P.

TECHNICAL NOTE

D-78

ANALYTICAL COMPARISON OF HYDRAZINE WITH PRIMARY PROPELLANTS AS THE TURBINE DRIVE FLUID FOR HYDROGEN-FLUORINE AND HYDROGEN-OXYGEN ALTITUDE STAGE ROCKETS

By William T. Wintucky

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

WASHINGTON October 1959

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TECHNICAL NOTE D-78

ANALYTICAL COMPARISON OF HYDRAZINE WITH PRIMARY PROPELLANTS AS THE TURBINE DRIVE FLUID FOR HYDROGEN-FLUORINE AND HYDROGEN-OXYGEN ALTITUDE STAGE ROCKETS

By William T. Wintucky

SUMMARY

Primary propellants, hydrogen-fluorine and hydrogen-oxygen, were compared with a high-energy monopropellant, hydrazine, as the turbine drive fluid for two high-energy-rocket altitude stages of 20,000 pounds nominal thrust. The effect of the two turbine drive systems on the payload was compared over a range of missions. The use of the monopropellant turbine drive system caused a penalty in stage payload of 45 to 65 pounds for the hydrogen-fluorine stage and 75 to 95 pounds for the hydrogen-oxygen stage. Thus, except for extreme missions where the payload itself becomes small, the monopropellant system is sufficiently competitive to the bleed system that its advantages from a control and starting standpoint might make the monopropellant system more desirable. The hydrazine turbine flow rate was about three times that required for the bleed systems for each propellant combination because of the difference in specific heats of the fluids. The hydrogen-fluorine stage required about one-half the turbine flow rate of the hydrogen-oxygen stage because of the difference in turbine work requirements.

INTRODUCTION

In recent years considerable interest has been shown in the use of high-energy propellants for improving the payload capability of upper rocket stages. The use of such high-energy propellants as hydrogen-fluorine and hydrogen-oxygen appears attractive for this type of stage because of the increased specific impulse.

In the design of these rocket stages, simplification of the control and starting systems of the engine is desirable. One way of accomplishing this would be to use a separate monopropellant as the turbine drive fluid instead of the main propellants. However, a problem exists for these high-energy-rocket applications in that monopropellants, in general, require an increase in flow rate through the turbine. The increased flow rate, since it represents a reduction in the effective specific impulse, results in a reduction in the payload. The significance of this difference in flow rate between the two turbine drive systems must first be determined in terms of payload penalty before the other aspects are investigated.

Two rocket stages were selected, one using hydrogen-fluorine and the other hydrogen-oxygen. For each stage, two turbine drive systems were compared: the monopropellant drive system, and a bleed system using the fluids from the pump outlets. The systems were compared for the upper rocket stages having a fixed gross weight of 13,333 pounds and a nominal thrust of 20,000 pounds. This report presents the results of the study showing the relative effect of the two turbine drive systems used for the two rocket stages in terms of the influence on stage payload.

DESCRIPTION OF ROCKET SYSTEMS

The rocket stage and turbine drive systems considered in this report are shown in figure 1. The rockets are high-energy stages designed for altitude operation and consist primarily of the thrust-chamber section, turbopump section, tankage section, and the payload, which includes the nose cone and any guidance of the stage. Also shown in detail is the turbopump; the inserts show the components and arrangement of the two turbine drive systems that are compared in this report.

The turbopump consists of the fuel and oxidant pumps, with the fuel pump directly coupled to the turbine with gearing to the oxidant pump. This system is desirable, since the low density of the hydrogen allows the hydrogen pump to be operated at high rotative speeds (see ref. 1).

The turbine drive system shown in the upper right corner of figure 1 is called the "bleed system." A small percentage of the main pump flow is bled off from the pump exits, burned in a gas generator passed through the turbine, and subsequently exhausted overboard. The second turbine drive system, shown in the lower right part of figure 1, is referred to herein as the "monopropellant system." This system consists of a third fluid (hydrazine) stored in a separate tank pressurized from a separate helium tank. The hydrazine is passed through a decomposition chamber, the products of which drive the turbine and then are exhausted overboard. For both turbine drive systems, it is assumed that the turbine exhaust does not contribute to the thrust of the stage. The turbines shown are similar in size and stage number.

The rocket-stage features used in the analysis for the hydrogen-fluorine stage with the bleed system include:

Gross weight, W_g, lb 13,333
Engine thrust, F, lb 20,000
Initial acceleration, as a result of W_g and F 1.5

For the hydrogen-fluorine stage using the monopropellant system and the hydrogen-oxygen stage using both turbine drive systems, the engine thrust and initial acceleration were allowed to vary to maintain the same mission. (All symbols are defined in appendix A.)

Two high-energy propellant combinations are considered: hydrogen-fluorine and hydrogen-oxygen. The thrust-chamber specific impulse and percent fuel that are used in the analysis were selected as follows:

	I, sec	Percent fuel by weight
H_2-F_2	403	7
H_2-O_2	412	15

The specific-impulse values correspond to ideal frozen combustion in the thrust chamber: 300 pounds per square inch absolute chamber pressure at a pressure ratio of 800 to 1. This represents a nominal area ratio (nozzle-exit-area to throat-area ratio) of about 35 to 1. The specific-impulse values for hydrogen-fluorine came from unpublished analytical data obtained at Lewis Research Center, and the values for hydrogen-oxygen came from reference 2.

The following sections present the method used for determining the effect of the turbine drive fluid on the payload. Where numerical examples are required, they are presented for the hydrogen-fluorine stage using the bleed system.

METHOD OF ANALYSIS

Mission Considerations

The effect of the turbine drive fluid on the stage payload is considered over a range of missions hereafter referred to as "velocity increments" that are added by the stage during powered flight. These missions, for simplification of calculations, cover only vertical and horizontal powered-flight trajectories. No attempt is made to show what happens to the stage after burnout. Since the stage gross weight is specified, the empty weight W_e represents the dependent variable; this must be obtained before the payload can be determined.

Vertical mission. - The vertical mission is considered first, since the horizontal mission is merely a simplification. This mission represents both the attainment of a specified increment of velocity ΔV_B and a specified increment of altitude Δa_B. The equations used are:

Vertical-velocity-increment equation,

$$\Delta V_B = gI(1 - y)\left[\ln\frac{W_g}{W_e} - \frac{t_B}{I(1 - y)}\right] \tag{1}$$

Altitude-increment equation,

$$\Delta a_B = t_B\left[gI(1 - y) - \frac{\Delta V_B + gt_B}{(W_g/W_e) - 1} - \frac{1}{2}gt_B\right] \tag{2}$$

Initial-acceleration equation,

$$\frac{W_g}{W_e} = \left\{1 - G\left[\frac{t_B}{I(1 - y)}\right]\right\}^{-1} \tag{3}$$

Equations (1) and (3) are from reference 3, and equation (2) is from reference 4. Equations (1) and (2) neglect the effect of changing gravity with altitude, and a constant $g = 32.17$ feet per second per second is assumed along with no aerodynamic drag. The change of impulse with altitude is also neglected. The term y represents the turbine propellant flow rate as a ratio of the total pump flow rate w_T/w_P and causes a reduction in effective specific impulse, assuming no thrust recovery.

In order to determine the vertical mission, the range of velocity increments covered was from 10,000 to 25,000 feet per second. The value of G was selected at 1.5. The corresponding altitude increment was computed for the hydrogen-fluorine stage using the bleed system, with $y = 0.00325$ (this number was obtained from turbine calculations discussed later.) The calculation first involved a simultaneous solution of equations (1) and (3), and then a direct solution of equation (2). The results of these calculations are presented in figure 2. The range of velocity increments considered (10,000 to 25,000 ft/sec) resulted in altitude increments of approximately 600,000 to 1,500,000 feet. The stage empty weight can be computed from equation (3). The stage empty weight for the hydrogen-fluorine bleed system is shown as the lower curve in figure 3.

E-231

The conditions shown in figure 2 define the vertical mission for all four stages. For the hydrogen-fluorine stage using the monopropellant system and the hydrogen-oxygen stage using both drive systems, these conditions are used in a simultaneous solution of equations (1) and (2). The acceleration parameter G was then allowed to vary; this resulted in the thrust deviating slightly from the 20,000-pound original value, since the gross weight of the stage was assumed to be fixed. The value of G was computed to obtain the thrust. From this solution, the required empty weight was computed.

Horizontal mission. - The horizontal mission is defined simply as a velocity increment, and equation (1) reduces to

$$\Delta V_B = gI(1 - y)\ln \frac{W_g}{W_e} \tag{4}$$

The empty weight was then computed directly from equation (4) with the same y as in the vertical mission; the upper curve in figure 3 shows this for the hydrogen-fluorine bleed system. The burning time required in future calculations was then computed from equation (3) by using the value of G obtained for the vertical case at the same velocity increment. As a result, the same thrust-chamber weight flow is used in both the vertical and horizontal flight calculations for a given velocity increment. Once the empty weight is known, the payload can then be obtained through consideration of the stage component weights.

Payload Considerations

The calculation of the payload P for a given velocity increment requires a knowledge of the weights of the various structural components. This involves arbitrary gross assumptions that determine the level of the calculated numbers. The assumptions are the same for all examples, so that the trends are valid although the absolute values may be in question. Some component weights are kept constant while others are varied. An equation for the payload can be written as:

$$P = W_e - W_{th} - W_{TP} - W_{tank} - W_{outage} - W_{mono} - W_{misc} \tag{5}$$

The empty weight W_e was obtained from the mission calculations. The method for obtaining the component weights is described in the following paragraphs.

Thrust-chamber weight. - The thrust-chamber weight W_{th} is assumed to include the weights of the thrust chamber as well as lines, valves, and controls associated with the thrust-chamber flow. For a 20,000-pound-nominal-thrust, regeneratively cooled unit of the type considered herein, the weight W_{th} was estimated to be 240 pounds.

Turbopump weight. - The turbopump unit includes the turbine, pumps, gearing, gas generator or decomposition chamber, and accessories. The turbopump weight W_{TP} was assumed to be constant for all the drive systems considered. This weight was estimated to be 110 pounds.

Main-tanks weight. - The main-tanks weight consists of the fuel tank, oxidant tank, and helium tank. These weights vary with the propellant combination and the amount of liquid required. The tank weight W_{tank} must be computed for each mission and requires a knowledge of the propellant volumes. The propellant volumes are calculated from the pump flow rate and the burning time for the particular mission. For the stages using a bleed turbine system, the pump flow rate is the sum of the thrust-chamber flow rate and turbine flow rate. The calculation of the pump fuel-flow rate requires the use of the percent fuel in both the thrust chamber and the turbine gas generator; that is,

$$w_f = w_{f,th} + w_{f,T}$$

$$w_f = w_{th}\left(\frac{w_f}{w_P}\right)_{th} + w_T\left(\frac{w_f}{w_P}\right)_T \qquad (6)$$

Since the percent fuel $(w_f/w_P)_{th}$ for the thrust chamber has been specified and the percent fuel through the turbine $(w_f/w_P)_T$ is specified in the following table, the total fuel-flow rate can be computed.

System	Turbine fluid flow (by weight)	
	Percent fuel	Percent oxidant
H_2-F_2	55	45
H_2-O_2	57	43

When the total fuel-flow rate is known, the oxidant flow rate can be computed by merely subtracting the total fuel-flow rate from the pump flow rate. The total weight of each propellant used is then the product of the total individual flow rate times the burning time. From the total weights of each propellant, the volume required can be obtained.

The propellant tanks designed to hold the required propellant volumes were evolved from the following assumptions:

E-231

Diameter of propellant tanks, ft	5
Tank ends (see fig. 1)	spherical
Tank gas pressure, lb/sq in. abs (for necessary rigidity, tanks are assumed to be main supporting members of stage)	constant at 100
Ullage, percent of tank volume	10
Outage, percent of propellants by weight	1
Tank-wall working stress, psi	100,000
Metal density, lb/cu ft	500
Helium-pressurizing tank submerged in oxidant tank	
Tank lengths changed to accommodate different volumes	

For the monopropellant system, the thrust-chamber weight flow is equal to the pump weight flow, $w_p = w_{th}$, since no pump fluid is used to drive the turbine. The individual propellant weight is, then, just the pump weight flow times the burning time, multiplied by either the percent fuel or oxidant of the thrust chamber.

The helium tank used to pressurize the main tanks was assumed to have the following features:

Diameter, ft	3
Tank ends	spherical
Initial gas pressure, lb/sq in. abs	1000
Minimum gas pressure, lb/sq in. abs	100
Tank-wall working stress, psi	100,000
Metal density, lb/cu ft	500
Volume required determined from main-tank volumes	
Tank length changed to accommodate different volumes	

The monopropellant fluid flow rate is simply y times the thrust-chamber weight flow rate, since $w_p = w_{th}$. The total weight of hydrazine used is obtained from

$$W_{N_2H_4} = y w_p t_B \tag{7}$$

The weights of the hydrazine and pressurizing tanks were obtained in a manner similar to the main tanks, except that spherical tanks were used. The ullage, outage, tank wall stress, and metal density were the same. The initial pressure of this helium tank was 1500 pounds per square inch absolute, and pressure at the end of burning time was 350 pounds per square inch absolute.

Miscellaneous weights. - Miscellaneous weights W_{misc} were assumed to include extra skin, thrust cage, gimbaling, tank supports, and tank baffles. From cursory studies, the sum of these weights was estimated to be 345 pounds.

From consideration of these weights, some of which are fixed and some of which are varied, the stage payload can be calculated. Figure 4 shows the breakdown of equation (5) for the hydrogen-fluorine vertical-flight case where component weight is a function of velocity increment.

Turbine Considerations

The turbine weight was assumed to be the same for the different fluids and is not included as a variable. The primary effect of the turbine design is that of the different turbine flow rates on the effective specific impulse. This effect has already been described previously. This section discusses the method for obtaining the necessary curves from which the bleed rates can be selected. The method considers the bleed rate y and the turbine stage number n as independent variables. Final selection of the turbine design is made from a consideration of:

(1) Flow rate

(2) Turbine-inlet pressure

(3) First-stage stator blade height

The method of obtaining the turbine designs for screening is the same as that of reference 3 with the exception of certain modifications that are described as they arise.

Determination of turbine power requirement. - The turbine power requirement was obtained on a per-unit pump flow basis. The following summarizes the assumptions used to make the calculations:

Pressure rise across the pumps, $\Delta p_f = \Delta p_o$, lb/sq ft 72,000
Pump efficiency, $\eta_{P,f} = \eta_{P,o}$ 0.65

The resulting work required of the turbine per unit pump flow rate was computed by using equation (16) of reference 3. The calculated results are:

For H_2-F_2,

$$\Delta H'/w_P = 3.66 \text{ Btu/lb}$$

For H_2-O_2,

$$\Delta H'/w_P = 6.53 \text{ Btu/lb}$$

Considerable difference exists in the work required by the two propellant combinations because of the considerably different percent fuel used in the two combinations, as well as the difference in densities of the oxidants.

Determination of turbine frontal area parameter. - The turbine frontal area parameter A_F/w_P was obtained by using the following turbine design features:

Rotative speed, rpm 50,000
Tip speed, U_t, ft/sec 1,400

This parameter can be computed from the following equation, which is a rearranged form of equation (19) of reference 3:

$$\frac{A_F}{w_P} = \left(\frac{30\ U_t}{\text{rpm}}\right)^2 \frac{1}{\pi w_P} \tag{8}$$

The pump flow rate used in these calculations was actually the thrust-chamber flow rate used in the mission calculations. For the monopropellant system this is exactly the pump flow rate. However, for the bleed system this is in error by the amount of the bleed rate, since the turbine flow rate is not included.

Required input data for turbine calculations. - In order to use the method of reference 3, additional input data were required; these are summarized in the following table:

System	c_p	γ	R	T_1'	$\left(\frac{V_x}{V_{cr}}\right)_3$	$\left(\frac{r_h}{r_t}\right)_3$
H_2-F_2	2.12	1.37	451	1860	0.4	0.8
H_2-O_2	2.13	1.36	438	↓	↓	↓
N_2H_4	.68	1.28	117	↓	↓	↓

The gas properties listed in the table represent those at the turbine inlet.

Calculation of turbine-inlet pressure. - The turbine-inlet pressure is calculated from a slightly rearranged form of equation (32) of reference 3:

$$p_1' = \frac{y\sqrt{T_1'}\left(1 - \frac{\Delta H'/w_P}{yc_pT_1'}\right)^{1/2}\left(\frac{T}{T'}\right)_3}{\frac{A_F}{w_P}\sqrt{\frac{2\gamma}{\gamma + 1}\frac{g}{R}}\left(\frac{V_x}{V_{cr}}\right)_3\left[1 - \left(\frac{r_h}{r_t}\right)_3^2\right]\frac{p_3}{p_1'}} \tag{9}$$

The two unknowns, temperature ratio and pressure ratio, in equation (9) can be calculated by using the method of reference 3.

Calculation of first-stage stator blade height. - One limitation that can become severe for high-pressure-ratio, low-flow turbines is the first-stage stator blade height. This must be considered in selecting a turbine design. Appendix B presents the method used for determining this blade height as a function of the turbine design parameters of reference 3. The calculations were made by using equation (B1) and the following assumptions:

First-stage stator-exit flow angle, α . . . . .15° from tangential
First-stage stator exit-to-inlet total-pressure ratio, L 0.90

In selecting a turbine design for each case, a lower limit of 0.2 inch for stator-exit blade height is arbitrarily used from consideration of losses and fabrication.

RESULTS OF ANALYSIS

Turbine Parameters

Results of the calculations of the various turbine parameters using the method of reference 3 are shown in figures 5 and 6 for the hydrogen-fluorine and hydrogen-oxygen stages, respectively, with each figure covering both the turbine bleed and monopropellant systems. Shown are curves of a given turbine-stage number over a range of inlet pressures and bleed rates. Lines of constant inlet blade height are also shown.

In general, for a given turbine-stage number, the effect of reducing the bleed rate is to increase the required inlet pressure level. The effect of increasing the number of turbine stages is to reduce the level of bleed rate. A minimum bleed rate necessitates an increased number of turbine stages. For the purpose of this analysis, three stages were arbitrarily selected as a compromise between efficiency and complexity. In going to increased pressures, the problem of small inlet blade heights is encountered very quickly so that, when the limitation of 0.2 inch inlet blade height is imposed, the following bleed rates and inlet pressures result:

E-231

Stage	H_2-F_2		H_2-O_2	
	Bleed	Monopropellant	Bleed	Monopropellant
Bleed rate, y	0.00325	0.00910	0.00590	0.01625
Inlet p_1', lb/sq in. abs	48	68	80	116

For the hydrogen-fluorine stage, the flow rates for the bleed and monopropellant systems were 0.00325 and 0.00910, respectively. Points to be noted are:

(1) The flow rate for the bleed system is low.

(2) The ratio of the flow rate of the monopropellant system to the bleed system is approximately 3 to 1, approximately the ratio of the specific heats of the turbine drive fluids.

The selection of an inlet blade height of 0.2 inch sets an area ratio that corresponds approximately to a pressure ratio of 10 to 1 across the turbine.

For the hydrogen-oxygen stage, the flow rates for the bleed and monopropellant systems were 0.00590 and 0.01625, respectively. The pertinent observations are:

(1) The ratio of the flow rate of the monopropellant system to the bleed system is again approximately 3 to 1, the ratio of the specific heats of the turbine drive fluids.

(2) The bleed rate of the hydrogen-oxygen stage is about twice that of the hydrogen-fluorine stage, which was of the same order as the ratio of the work requirements for the stages.

Rocket-Stage Payload

The rocket-stage payloads were obtained by using the bleed rates described in the preceding section. The payload-calculation results for the hydrogen-fluorine and hydrogen-oxygen stages are presented in figure 7. Shown is stage payload as a function of mission (velocity increment for vertical or horizontal flight) for the bleed and monopropellant turbine drive systems. As the mission becomes more difficult, the payload drops off rapidly. The vertical mission is more critical, since gravity is working against the stage.

For a given mission for the hydrogen-fluorine stage, a 45- to 65-pound reduction in payload is effected by using a monopropellant turbine system. The monopropellant system for the hydrogen-oxygen stage has about a 75- to 95-pound reduction in payload because of the difference in turbine flow rate. Thus, except for extreme missions where the payload itself becomes small, the monopropellant system is competitive to the point where its advantages from a control and starting standpoint might make the system more desirable than the bleed system.

SUMMARY OF RESULTS

This report presents a study of the effect of two turbine drive systems on the payload of two altitude rocket stages of approximately 20,000 pounds thrust using hydrogen-fluorine and hydrogen-oxygen as the propellants. The two drive systems included use of the primary propellants with a high-energy monopropellant, hydrazine, as the drive fluid. The results can be summarized as follows:

1. The use of hydrazine as a turbine drive fluid required approximately three times the flow rate of the bleed system because of the difference in specific heats. The hydrogen-fluorine rocket stage required low flow rates, about one-half that of the hydrogen-oxygen stage; this was due to the difference in turbine work requirements.

2. The effect of the two turbine systems on the payloads of the hydrogen-fluorine and hydrogen-oxygen stages was very small. The use of the monopropellant turbine drive system caused a reduction in payload (as compared with the bleed system) of 45 to 65 pounds for the hydrogen-fluorine stage and 75 to 95 pounds for the hydrogen-oxygen stage. Thus, except for extreme missions where the payload itself becomes small, the monopropellant system is sufficiently competitive to the bleed system that its advantages from a control and starting standpoint might make the monopropellant system more desirable.

CONCLUDING REMARKS

The analysis was made primarily to give an indication of the relative significance of the two turbine drive systems for the two high-energy rocket stages. The relative merits of the hydrogen-fluorine and hydrogen-oxygen stages should not be determined from these results, because they would involve a more accurate assumption of the specific impulse as well as a more accurate weight determination of the individual systems. The conclusions resulting from the comparison of the bleed system with the monopropellant system are considered valid, even though the results are a function of the assumptions made.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, June 23, 1959

APPENDIX A

SYMBOLS

The following symbols are used in this report.

A	area, sq ft
a	altitude, ft
c_p	specific heat at constant pressure
F	thrust, lb
G	ratio of thrust to gross weight
g	acceleration due to gravity, 32.17 ft/sec^2
$\Delta H'$	total enthalpy requirement of turbine, Btu/sec
h	blade height, ft
I	specific impulse, sec
J	mechanical equivalent of heat, 778.2 ft-lb/Btu
L	loss total-pressure ratio across stator, p_2'/p_1'
n	number of turbine stages
P	payload, lb
p	pressure, lb/sq ft
R	gas constant, ft/$^{\circ}$R
r	radius, ft
rpm	revolutions per minute
T	temperature, $^{\circ}$R
t	burning time, sec
U	turbine blade speed, ft/sec
V	absolute gas velocity, ft/sec

E-231

W	weight, lb
w	weight-flow rate, lb/sec
y	ratio of turbine flow to total pump flow (bleed rate)
α	flow angle measured from tangential direction, deg
γ	ratio of specific heats
Δ	increment
λ	speed-work parameter, $U_m^2/gJ\,\Delta h'$
ρ	density, lb/cu ft

Subscripts:

B	burnout, powered flight
cr	critical, corresponding to a flow Mach number of 1.0
e	empty
F	frontal area at turbine exit
f	fuel
g	gross
h	hub
m	mean
misc	miscellaneous
mono	monopropellant
outage	unusable fluid weight
o	oxidant
P	pump
ST	stage
T	turbine

tank	tanks
TP	turbopump
t	tip
th	thrust chamber
u	tangential component
x	axial
1	turbine inlet
2	exit at first stage stator
3	turbine exit

Superscript:

'	absolute total state

APPENDIX B

CALCULATION OF FIRST-STAGE STATOR BLADE HEIGHT

The first-stage stator blade height is represented by the height at the exit of the stator. Calculation of the height is made from continuity at the stator exit:

$$w_T = \rho_2 V_{x,2} A_2 = \rho_2 V_{x,2} 2\pi r_m h_2$$

Rearranging and introducing the pump flow rate gives

$$h_2 = \frac{(w_T/w_P)w_P}{\rho_2 V_{x,2} 2\pi r_m}$$

Substituting in the bleed rate for the ratio of turbine to pump flow rate and multiplying and dividing the denominator by $\rho_2' V_{cr}'$ result in

$$h_2 = \frac{y w_P}{\left(\dfrac{\rho V_x}{\rho' V_{cr}}\right)_2 \rho_2' V_{cr,2} 2\pi r_m}$$

Using the perfect gas law $\rho' = p'/RT'$ and the critical velocity relation $V_{cr} = \sqrt{\dfrac{2\gamma}{\gamma + 1} gRT'}$ results in the following expression for blade height:

$$h_2 = \frac{y w_P}{\left(\dfrac{\rho V_x}{\rho' V_{cr}}\right) \dfrac{p_1' L}{RT_2'} \left(\dfrac{2\gamma}{\gamma + 1} gRT_2'\right)^{1/2} 2\pi r_m} \tag{B1}$$

Since $T_2' = T_1'$, only one unknown exists in equation (B1): the mass-flow parameter $(\rho V_x/\rho' V_{cr})_2$, which is given in equation (B2). This is obtained by using the density ratio from the conservation of energy and the stator-exit axial velocity ratio:

$$\left(\frac{\rho V_x}{\rho' V_{cr}}\right)_2 = \left(\frac{V_x}{V_{cr}}\right)_2 \left\{1 - \frac{\gamma - 1}{\gamma + 1}\left[\left(\frac{V_x}{V_{cr}}\right)_2^2 + \left(\frac{V_u}{V_{cr}}\right)_2^2\right]\right\}^{1/(\gamma-1)} \tag{B2}$$

In order to calculate the mass-flow parameter, the turbine flow relations of reference 3 and the assumed stator angle are used. Since all stages are assumed to do equal work, the stage speed-work parameter is calculated from

$$\lambda_{ST} = n\lambda \tag{B3}$$

Also, in terms of diagram quantities,

$$\lambda_{ST} = \left(\frac{U}{\Delta V_u}\right)_m \tag{B4}$$

The mean-section rotor blade speed is

$$U_m = \frac{1}{2} U_{t,3} \left[1 + \left(\frac{r_h}{r_t}\right)_3\right]$$

The tangential velocity parameter $(V_u/\Delta V_u)_2$ out of the stator for impulse rotors is:

$$\left(\frac{V_u}{\Delta V_u}\right)_2 = \left(\lambda_{ST} + \frac{1}{2}\right) \tag{B5}$$

By using equations (B4) and (B5), the tangential velocity ratio out of the stator can be written as

$$\left(\frac{V_u}{V_{cr}}\right)_2 = \frac{U_m}{V_{cr,2}} \left(1 + \frac{1}{2\lambda_{ST}}\right) \tag{B6}$$

The stator-exit axial velocity ratio is computed from equation (B6) and the stator-exit flow angle:

$$\left(\frac{V_x}{V_{cr}}\right)_2 = \left(\frac{V_u}{V_{cr}}\right)_2 \tan \alpha_2 \tag{B7}$$

REFERENCES

1. Ginsburg, Ambrose, Stewart, Warner L., and Hartmann, Melvin J.: Turbopumps for High Energy Propellants. Paper No. 59-53, Inst. Aero. Sci., 1959.

2. Gordon, Sanford, and McBride, Bonnie J.: Theoretical Performance of Liquid Hydrogen with Liquid Oxygen as a Rocket Propellant. NASA MEMO 5-21-59E, 1959.

E-231

3. Stewart, Warner L., Evans, David G., and Whitney, Warren J.: A Method for Determining Turbine Design Characteristics for Rocket Turbodrive Applications. NACA RM E57K25a, 1958.

4. Rohlik, Harold E.: Analytical Investigation of the Significance of Turbine-Inlet Temperature in High-Energy Rocket Turbodrive Applications. NASA MEMO 1-6-59E, 1959.

E-231

CM-3 back

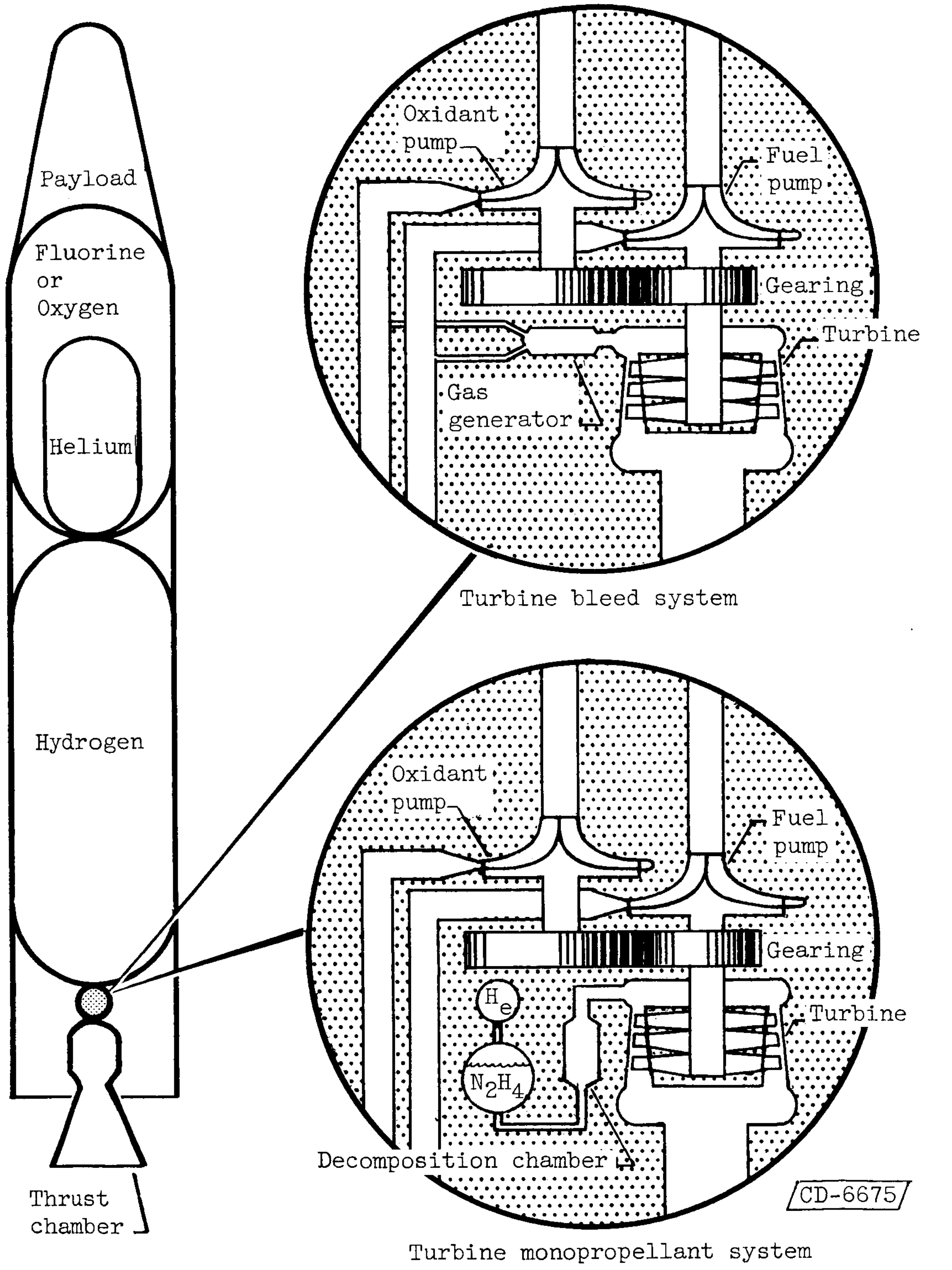


Figure 1. - Rocket stage showing the two turbopump systems considered.

E-251

E-231

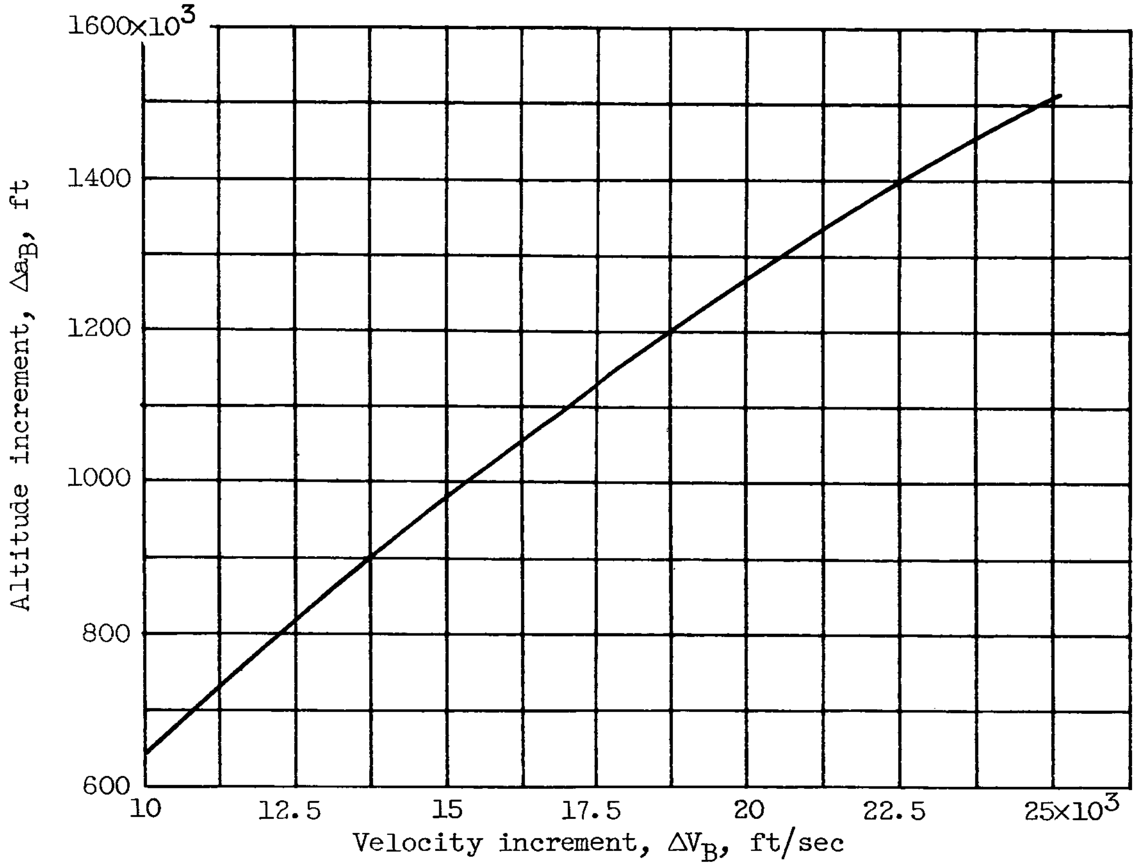


Figure 2. - Vertical mission as a function of altitude increment for a specified velocity increment.

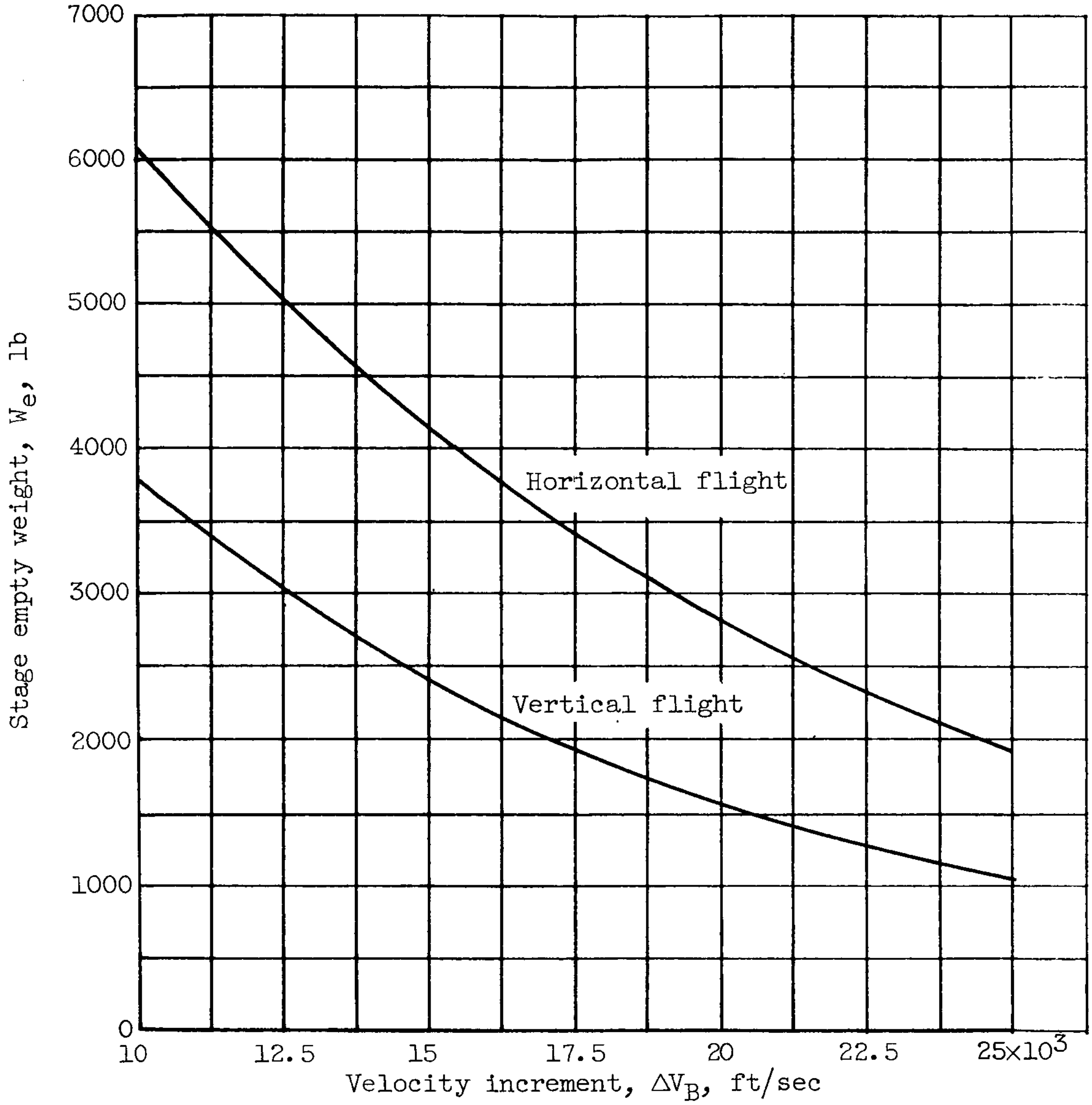
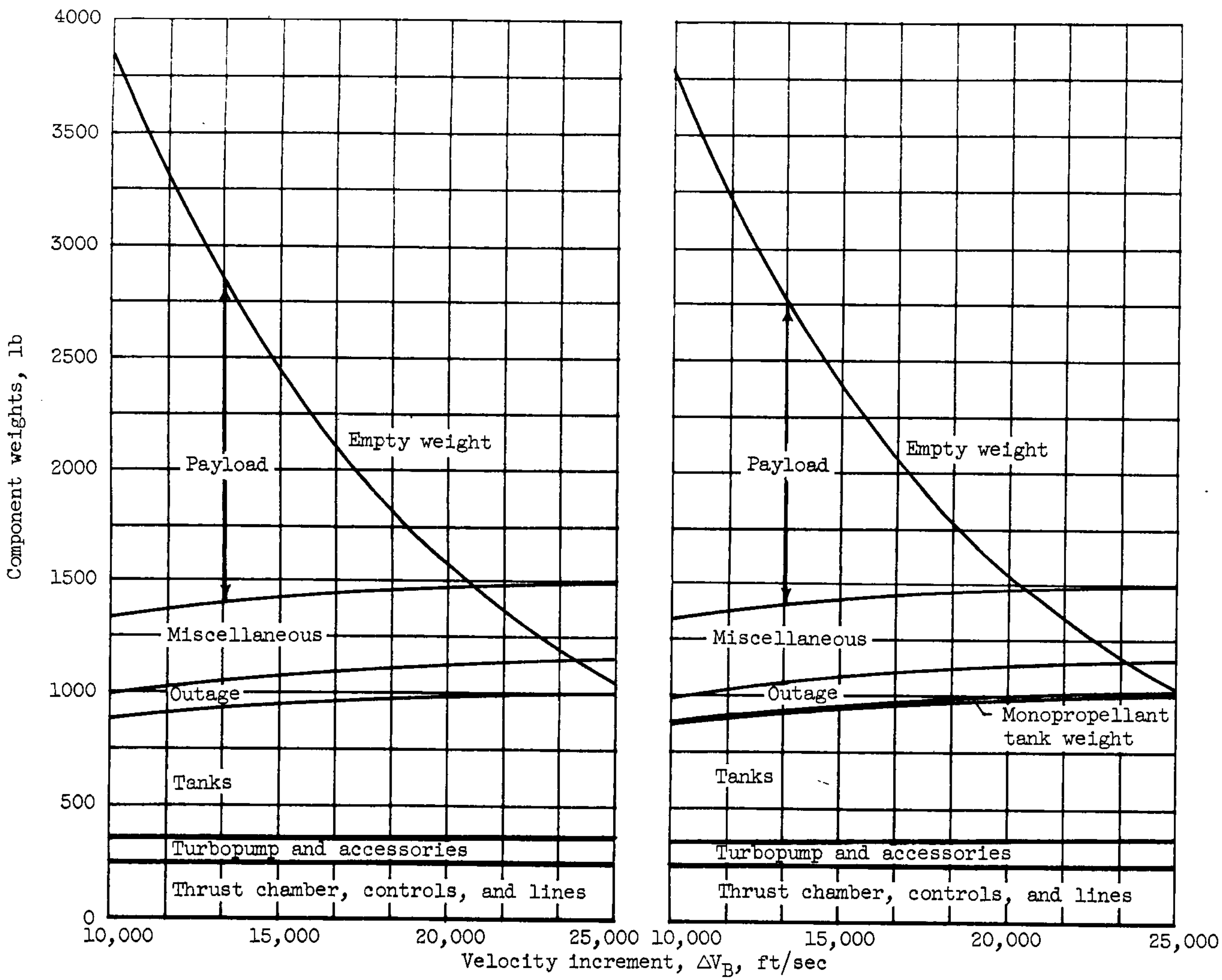


Figure 3. - Empty weight as a function of velocity increment for hydrogen-fluorine stage using bleed system.

E-231



(a) Bleed system stage.

(b) Monopropellant system stage.

Figure 4. - Component weights for hydrogen-fluorine stage as a function of velocity increments for vertical mission.

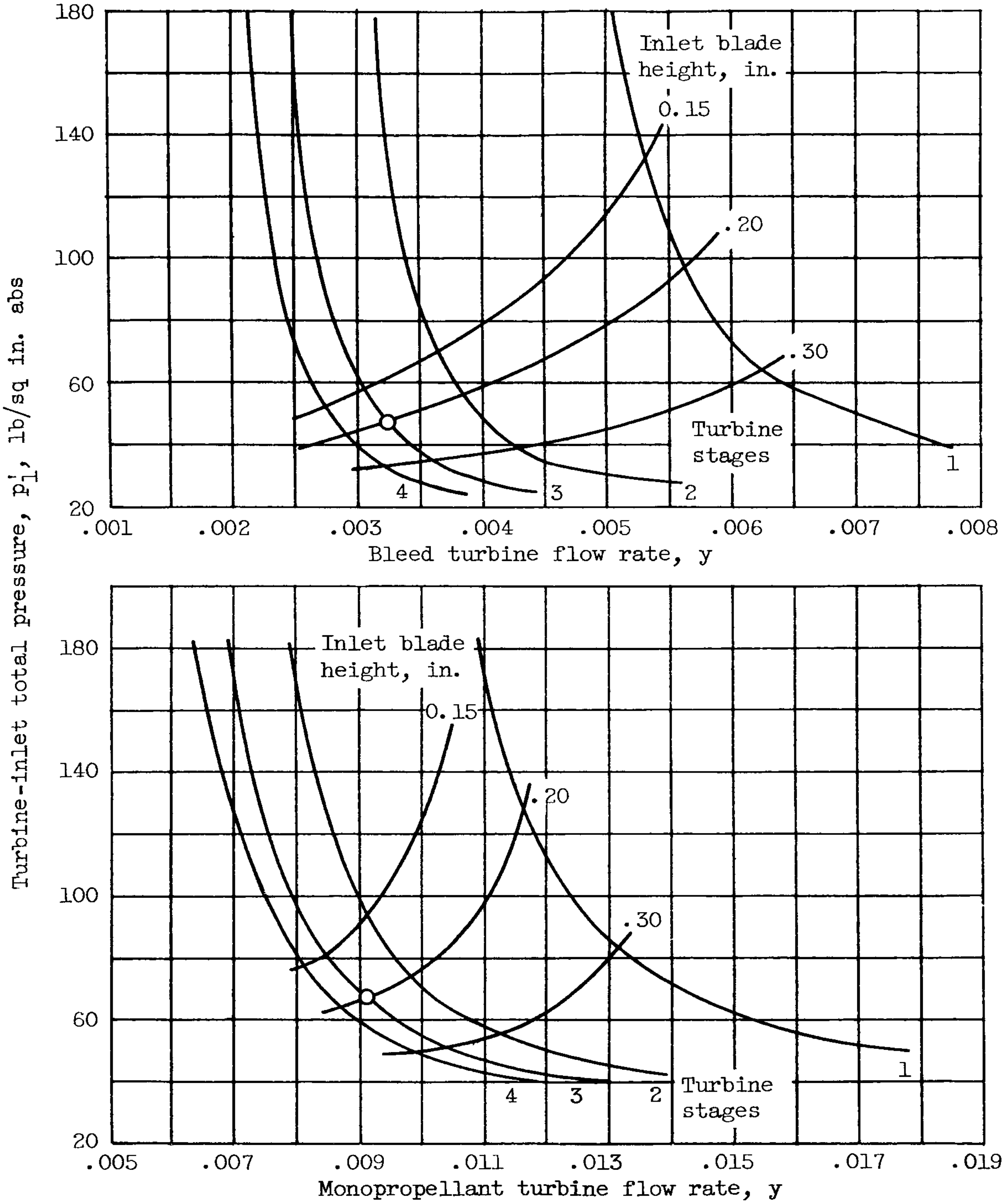


Figure 5. - Turbine-inlet pressure and blade height as a function of number of turbine stages and bleed rate for hydrogen-fluorine stage.

E-231

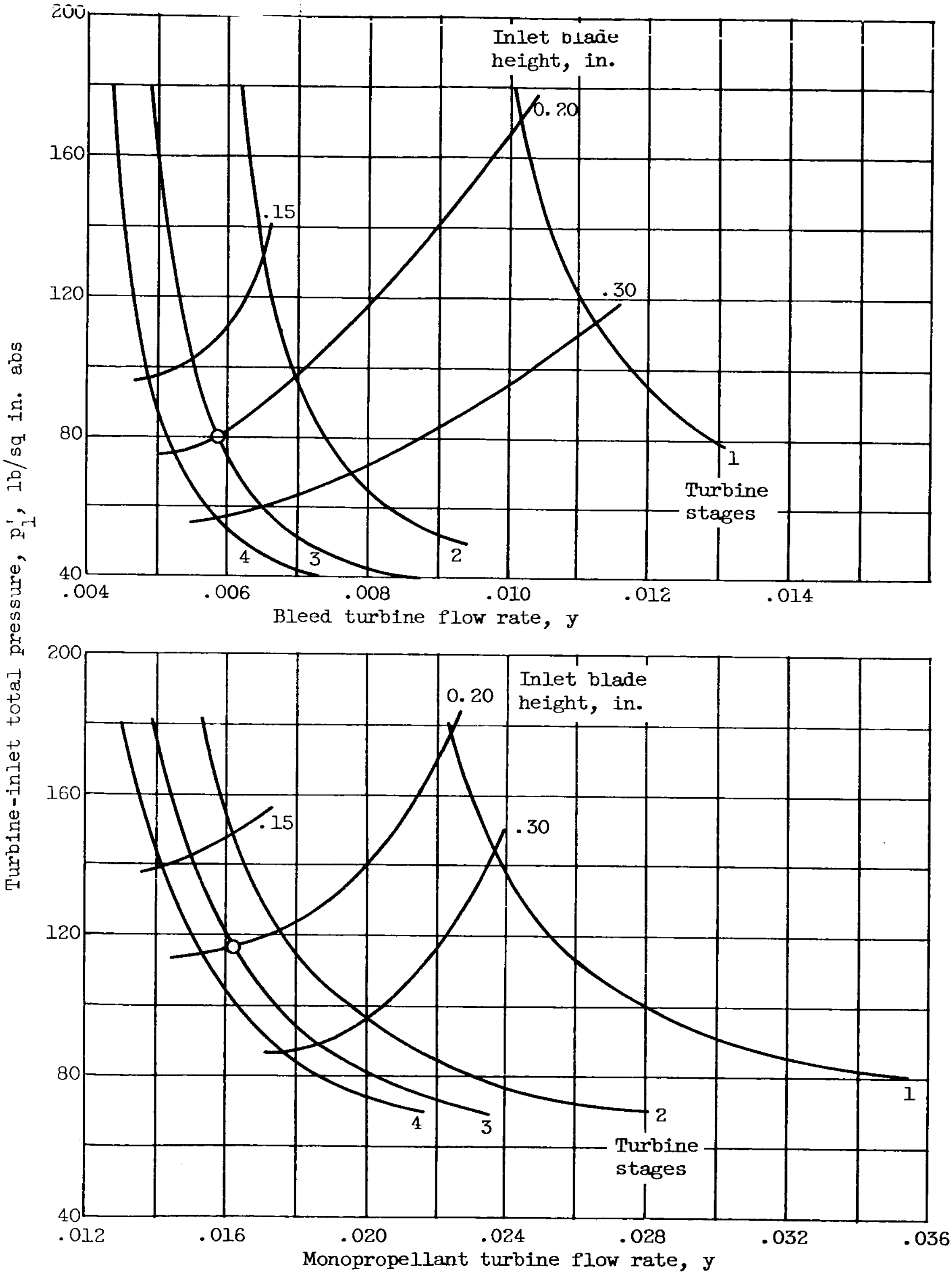


Figure 6. - Turbine-inlet pressure and blade height as a function of number of turbine stages and bleed rate for hydrogen-oxygen stage.

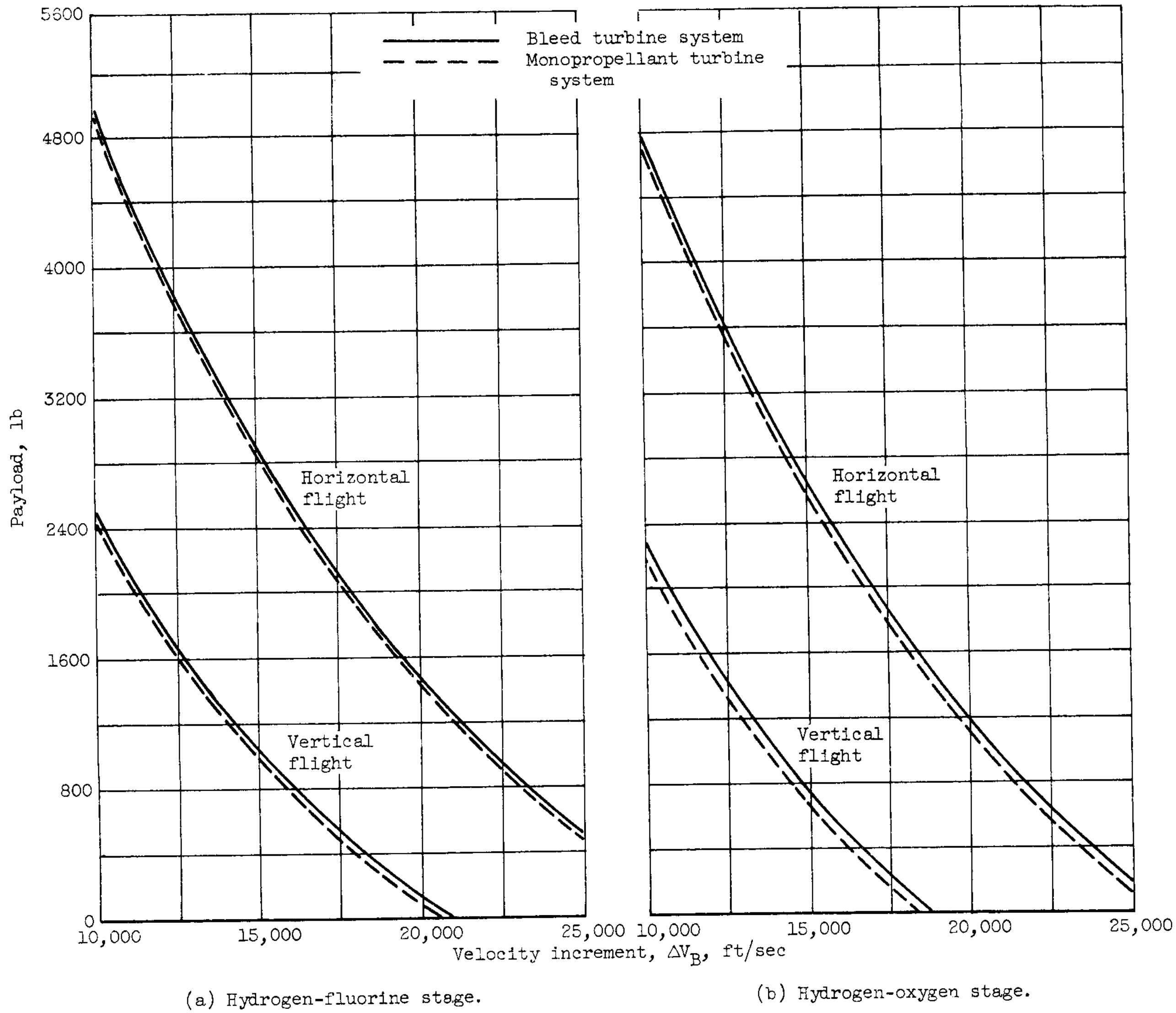


Figure 7. - Stage payload as a function of mission for bleed and monopropellant turbine systems.